Chair III: Database Systems
Chair XXV: Data Science and Engineering
Department of Informatics
Technical University of Munich

LLVM Code Optimisation for Automatic Differentiation

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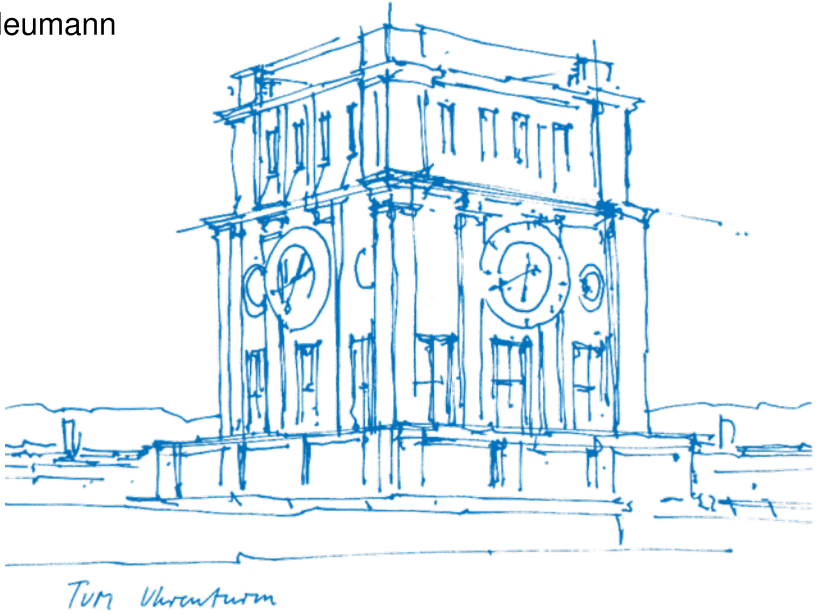
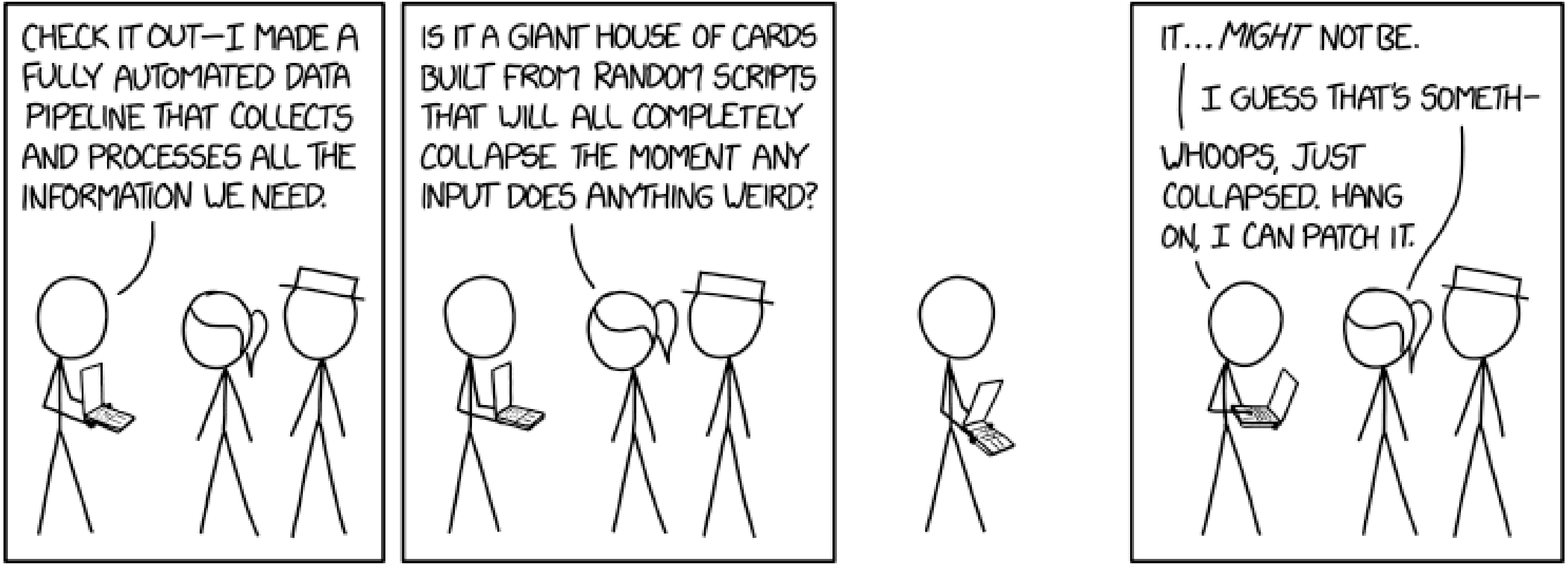
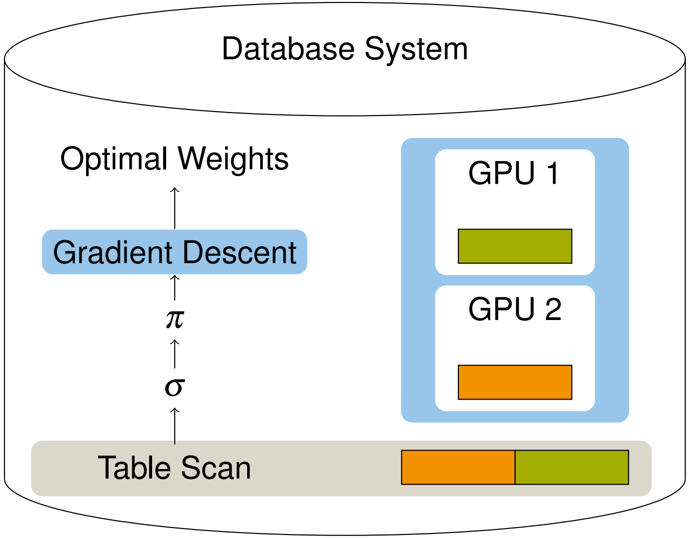


In-Database Machine Learning: Problem



In-Database Machine Learning: Solution

In-Database Machine Learning



- SQL sufficient for machine learning (ML)
  - Turing-complete with recursive tables
  - Sample operator for stochastic gradient descent
- Idea
  - Data preprocessing using SQL
  - No need for data extraction out of a database system
  - Label data within the database system using SQL

Structure

UMBRA

ML in SQL-92
Gradient descent with recursive tables

ML Operators
Automatic Differentiation

GPU support
Code-Generation for GPU

ML in SQL-92

ML in SQL-92: Gradient Descent with Recursive SQL

loss function $l_{x,y}(\vec{a,b})$, approximated values $m_{a,b}(x)$, given labels y, mean squared error:

$$m_{a,b}(x,y) = a \cdot x + b \approx y \tag{1}$$

$$l_{x,y}(a,b) = (a \cdot x + b - y)^2 \tag{2}$$

$$\nabla l_{x,y}(a,b) = \begin{pmatrix} \partial l / \partial a \\ \partial l / \partial b \end{pmatrix} = \begin{pmatrix} 2(ax+b-y) \cdot x \\ 2(ax+b-y) \end{pmatrix}. \tag{3}$$

Minimise $l_{x,y}(a,b)$: gradient descent (learning rate γ):

$$\begin{pmatrix} a_{t+1} \\ b_{t+1} \end{pmatrix} = \begin{pmatrix} a_t \\ b_t \end{pmatrix} - \gamma \nabla l_{x,y}(a_t, b_t), \tag{4}$$

$$\begin{pmatrix} a_\infty \\ b_\infty \end{pmatrix} \approx \lim_{t \to \infty} \left(\begin{pmatrix} a_t \\ b_t \end{pmatrix} \right). \tag{5}$$

```sql
create table data (x float, y float);
insert into data ...

with recursive gd (id, a, b) as (
 select 0,1::float,1::float
UNION ALL
 select id+1,
   a-0.05*avg(2*x*(a*x+b-y)),
   b-0.05*avg(2*(a*x+b-y))
 from gd, data
 where id<5 group by id,a,b)
select * from gd order by id;
```

Listing 1: Gradient descent.

Five iterations, loss function with two weights and $\gamma = 0.05$.

ML in SQL-92: Gradient Descent with Recursive SQL

loss function $l_{x,y}(\vec{a,b})$, approximated values $m_{a,b}(x)$, given labels y, mean squared error:

$$l_{x,y}(a,b) = (a \cdot x + b - y)^2 \tag{6}$$

$$\nabla l_{x,y}(a,b) = \begin{pmatrix} \partial l / \partial a \\ \partial l / \partial b \end{pmatrix} = \begin{pmatrix} 2(ax+b-y)\cdot x \\ 2(ax+b-y) \end{pmatrix}. \tag{7}$$

Minimise $l_{x,y}(a,b)$: gradient descent (learning rate γ):

$$\begin{pmatrix} a_{t+1} \\ b_{t+1} \end{pmatrix} = \begin{pmatrix} a_t \\ b_t \end{pmatrix} - \gamma \nabla l_{x,y}(a_t,b_t), \tag{8}$$

$$\begin{pmatrix} a_\infty \\ b_\infty \end{pmatrix} \approx \lim_{t\to\infty}\left(\begin{pmatrix} a_t \\ b_t \end{pmatrix}\right). \tag{9}$$

```sql
create table data (x float, y float);
insert into data ...

with recursive gd (id, a, b) as (
 select 0,1::float,1::float
UNION ALL
  select id+1, a-0.05*avg(d_a), b-0.05*avg(d_b)
  from umbra.derivation(TABLE (
    select id,a,b,x,y from gd,data where id<5),
    lambda (x) ((x.a * x.x + x.b - x.y)^2))
  group by id,a,b)
select * from gd order by id;
```

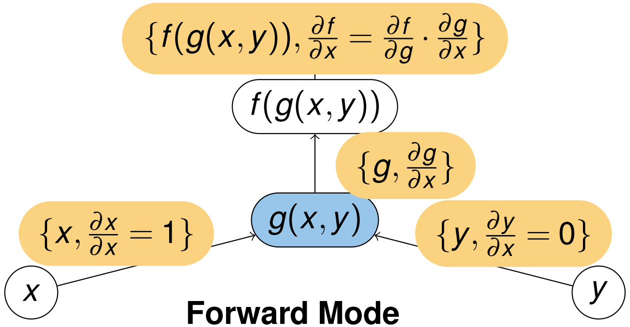
Listing 2: Gradient descent.

Five iterations, loss function with two weights and $\gamma = 0.05$.

ML Operators

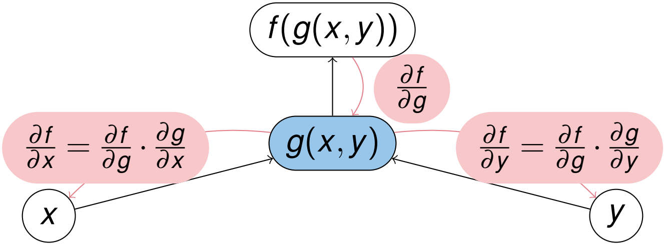
Forward or Reverse Mode Automatic Differentiation?

$$\frac{\partial f(g(x))}{\partial x} = \frac{\partial f}{\partial g} \cdot \frac{\partial g}{\partial x}$$



Forward Mode

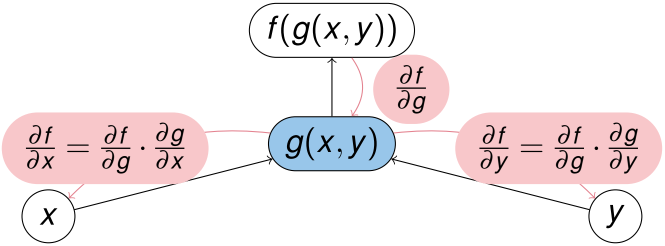
- requires one pass only (operator-overloading)
- leads to expression swell (?)



Reverse Mode

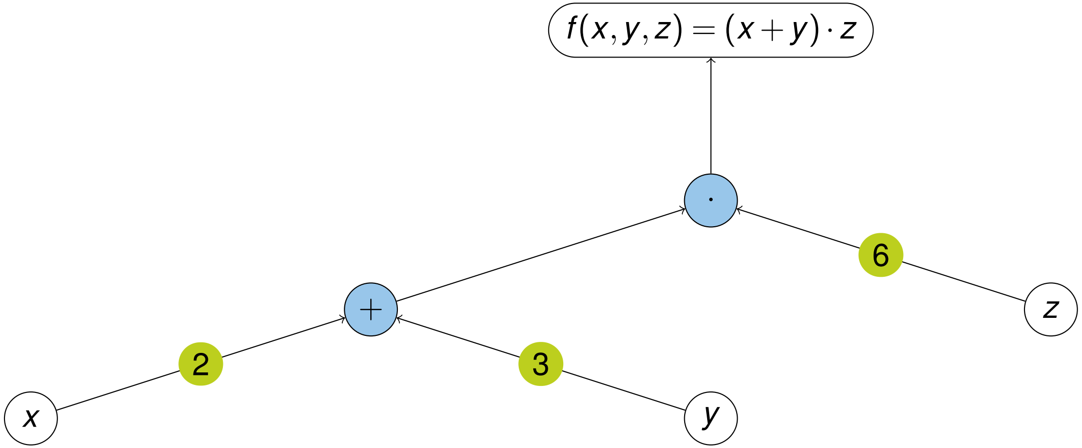
- computes all derivatives in one pass
- separate pass required

Reverse Mode Automatic Differentiation



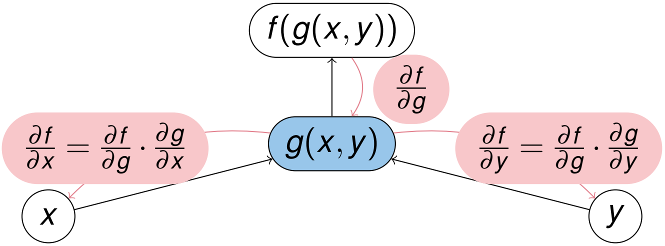
Automatic differentiation using reverse mode

- applying the chain rule to backpropagate the derivatives
- subexpressions are cached in LLVM registers for reuse




```
select * from umbra.derivation(
  TABLE(select 2 x,3 y,6 z),
  lambda(x)((x.x+x.y)*x.z));
-- x y z d_x d_y d_z
-- 2 3 6 6 6 5
```

Reverse Mode Automatic Differentiation



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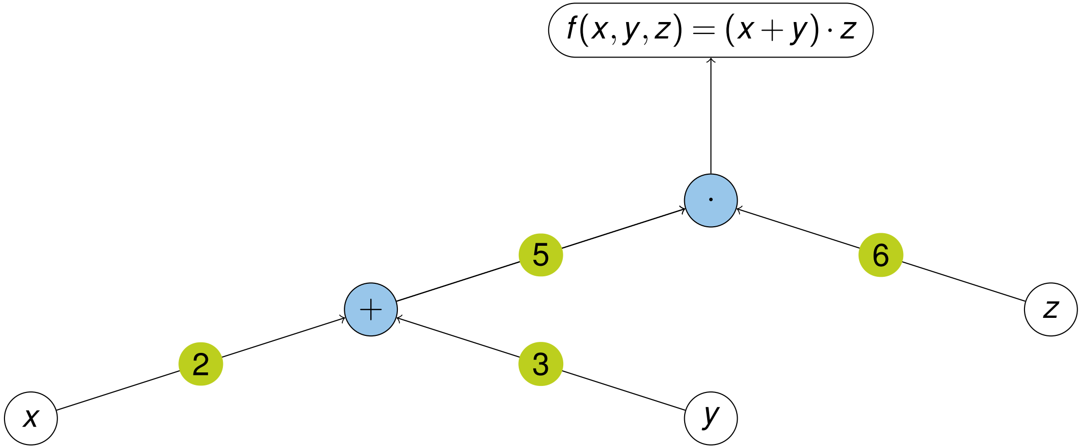


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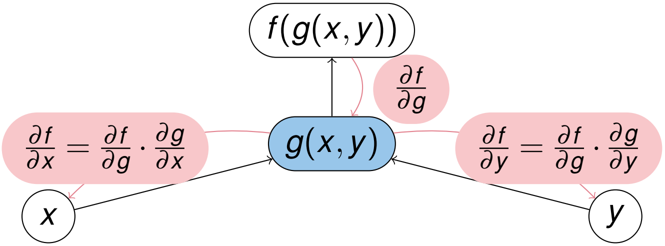

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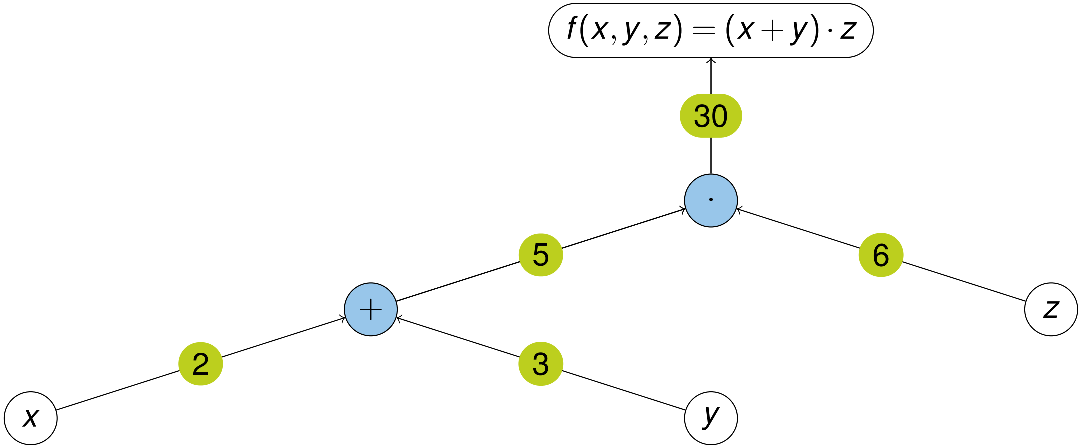


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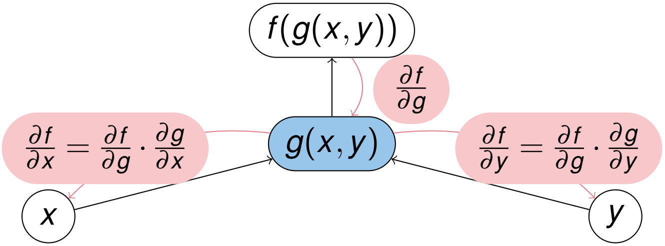

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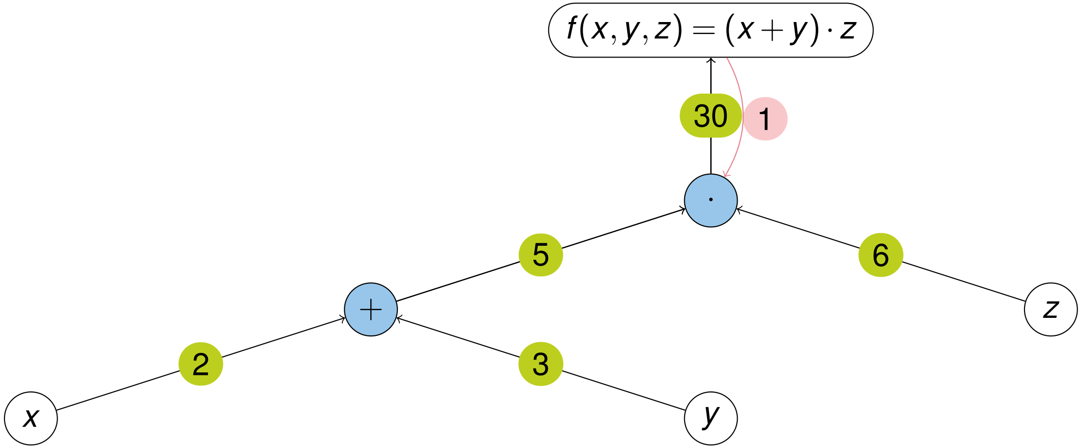


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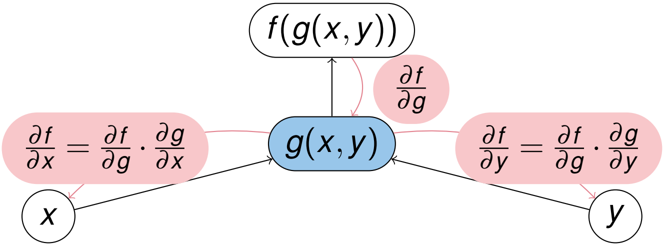
Reverse Mode Automatic Differentiation



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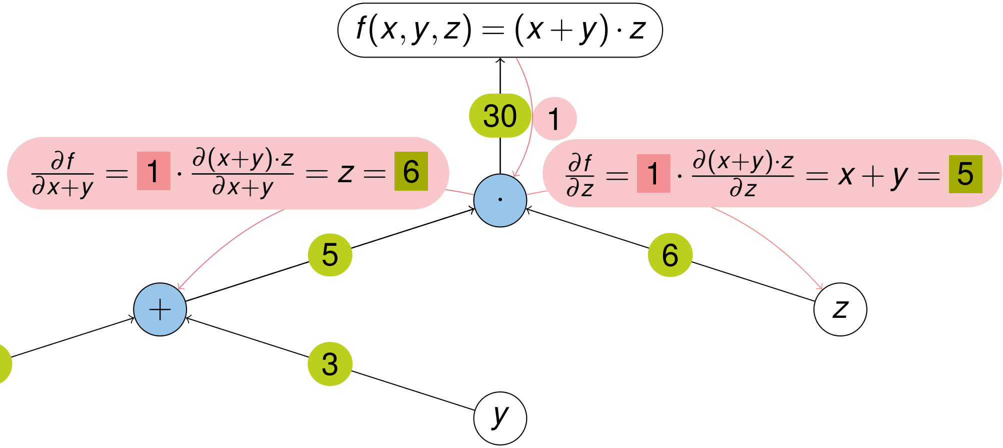


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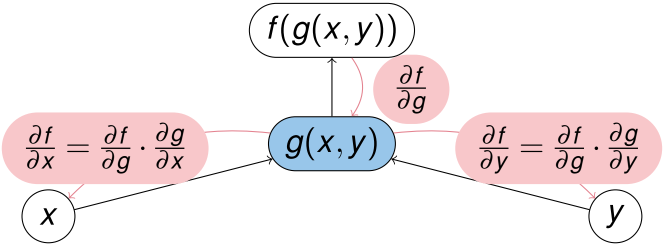
Reverse Mode Automatic Differentiation



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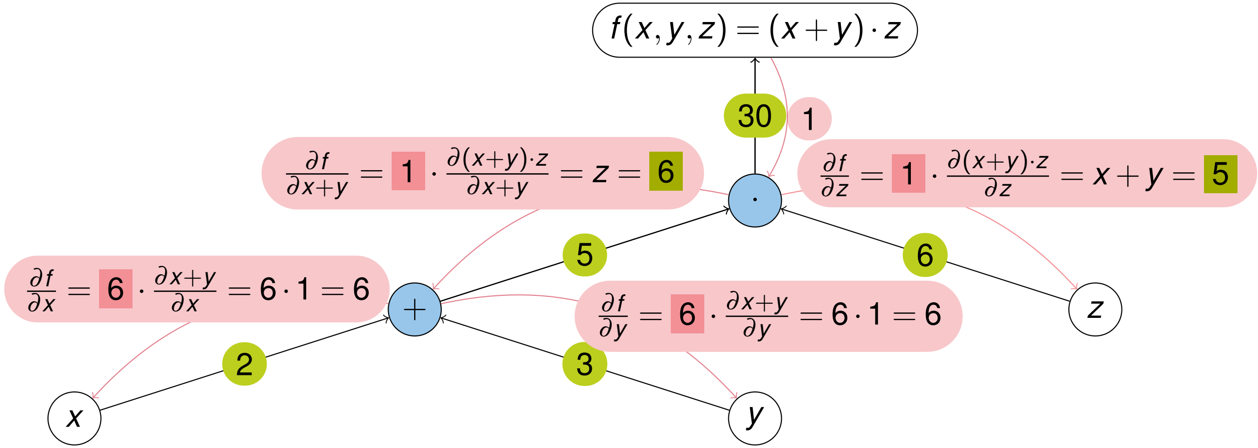


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select * from umbra.derivation(
  TABLE(select 2 x,3 y,6 z),
  lambda(x)((x.x+x.y)*x.z));
-- x y z d_x d_y d_z
-- 2 3 6 6 6 5
```

Reverse Mode Automatic Differentiation

function DERIVE(Z, *seed*)
  if Z matches $X + Y$ **then** DERIVE(X,*seed*); DERIVE(Y,*seed*)
  else if Z matches $X - Y$ **then** DERIVE(X,*seed*); DERIVE(Y,$-$*seed*)
  else if Z matches $X \cdot Y$ **then** DERIVE(X,$seed \cdot y$); DERIVE(Y,$seed \cdot x$)
  else if *isVariable*(Z) **then** $\frac{\partial}{\partial Z} \leftarrow \frac{\partial}{\partial Z} + seed$

Input: $(x+y)\cdot z$, $x := 2$, $y := 3$, $z := 6$
derive($(x+y)\cdot z$,1)

```
void deriveDerivation(/*...*/ Value seed, std::unordered_map<const IU*, Value>& derivatives) const {
 switch (z.getFunction()) {
  case KnownFunction::Add:
   context.deriveDerivatives(z.getLeft(), seed, derivatives);
   context.deriveDerivatives(z.getRight(), seed, derivatives);
   break;
  case KnownFunction::Mul:
   context.deriveDerivatives(z.getLeft(), seed.evaluateBinary(KnownFunction::Mul, right), derivatives);
   context.deriveDerivatives(z.getRight(), seed.evaluateBinary(KnownFunction::Mul, left), derivatives);
   break;
/*...*/
```

Reverse Mode Automatic Differentiation

function DERIVE(Z, *seed*)
  if Z matches $X + Y$ **then** DERIVE(X,*seed*); DERIVE(Y,*seed*)
  else if Z matches $X - Y$ **then** DERIVE(X,*seed*); DERIVE(Y,$-$*seed*)
  else if Z matches $X \cdot Y$ **then** DERIVE(X,$seed \cdot y$); DERIVE(Y,$seed \cdot x$)
  else if *isVariable*(Z) **then** $\frac{\partial}{\partial Z} \leftarrow \frac{\partial}{\partial Z} + seed$

Input: $(x+y)\cdot z$, $x := 2$, $y := 3$, $z := 6$
derive($(x+y)\cdot z$,1)

- derive($x+y$,$1 \cdot z = 1 \cdot 6$)
- derive(z,$1 \cdot (x+y) = 1 \cdot 5$)

```
void deriveDerivation(/*...*/ Value seed, std::unordered_map<const IU*, Value>& derivatives) const {
 switch (z.getFunction()) {
  case KnownFunction::Add:
   context.deriveDerivatives(z.getLeft(), seed, derivatives);
   context.deriveDerivatives(z.getRight(), seed, derivatives);
   break;
  case KnownFunction::Mul:
   context.deriveDerivatives(z.getLeft(), seed.evaluateBinary(KnownFunction::Mul, right), derivatives);
   context.deriveDerivatives(z.getRight(), seed.evaluateBinary(KnownFunction::Mul, left), derivatives);
   break;
/*...*/
```

Reverse Mode Automatic Differentiation

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  else if *isVariable*(Z) **then** $\frac{\partial}{\partial Z} \leftarrow \frac{\partial}{\partial Z} + seed$

Input: $(x+y)\cdot z$, $x := 2$, $y := 3$, $z := 6$
derive($(x+y)\cdot z$,1)

- derive($x+y$,$1\cdot z = 1\cdot 6$)
  - derive(x,$6\cdot 1 = 6$)
  - derive(y,$6\cdot 1 = 6$)
- derive(z,$1\cdot(x+y) = 1\cdot 5$)

```
void deriveDerivation(/*...*/ Value seed, std::unordered_map<const IU*, Value>& derivatives) const {
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   context.deriveDerivatives(z.getLeft(), seed.evaluateBinary(KnownFunction::Mul, right), derivatives);
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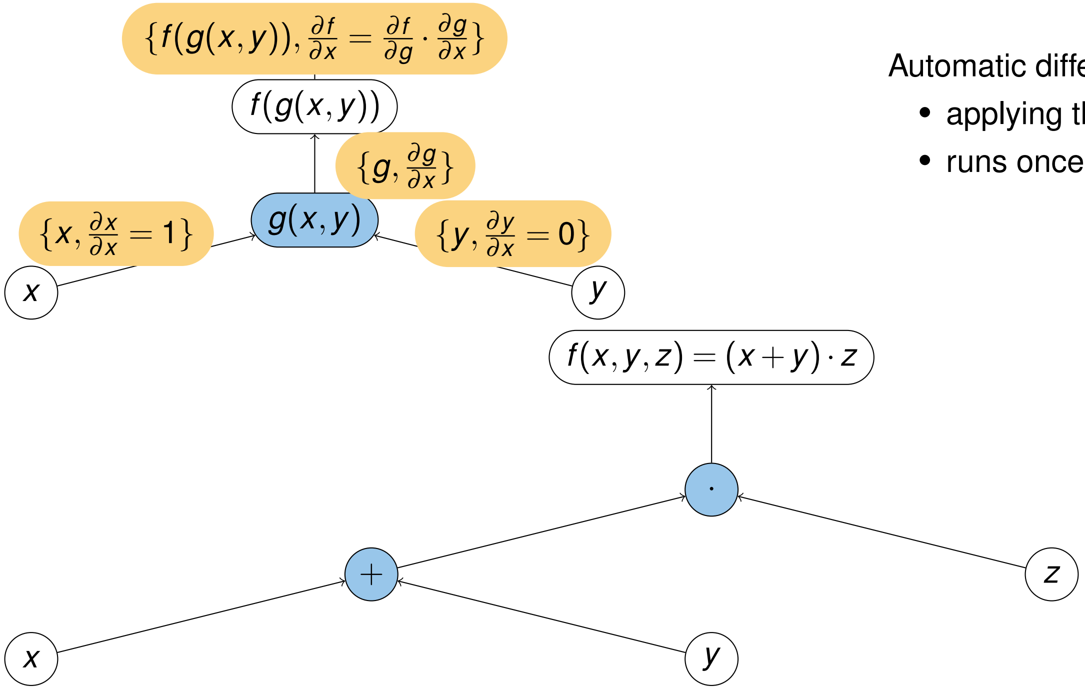
   break;
/*...*/
```

Forward Mode Automatic Differentiation

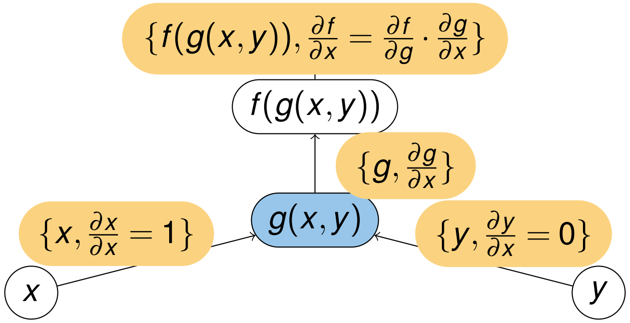


Automatic differentiation using forward mode

- applying the chain rule in the forward pass
- runs once per derivative

```
select * from umbra.derivation2(
  TABLE(select 2 x,3 y,6 z),
  lambda(x)((x.x+x.y)*x.z));
-- x y z d_x d_y d_z
-- 2 3 6 6 6 5
```

Forward Mode Automatic Differentiation



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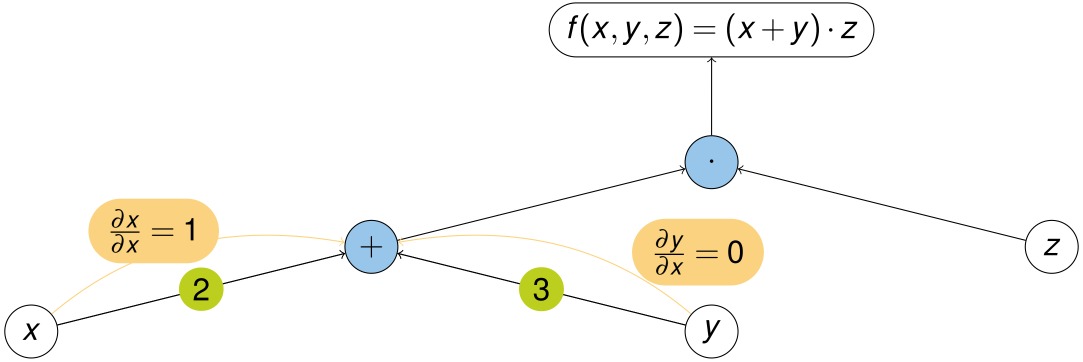


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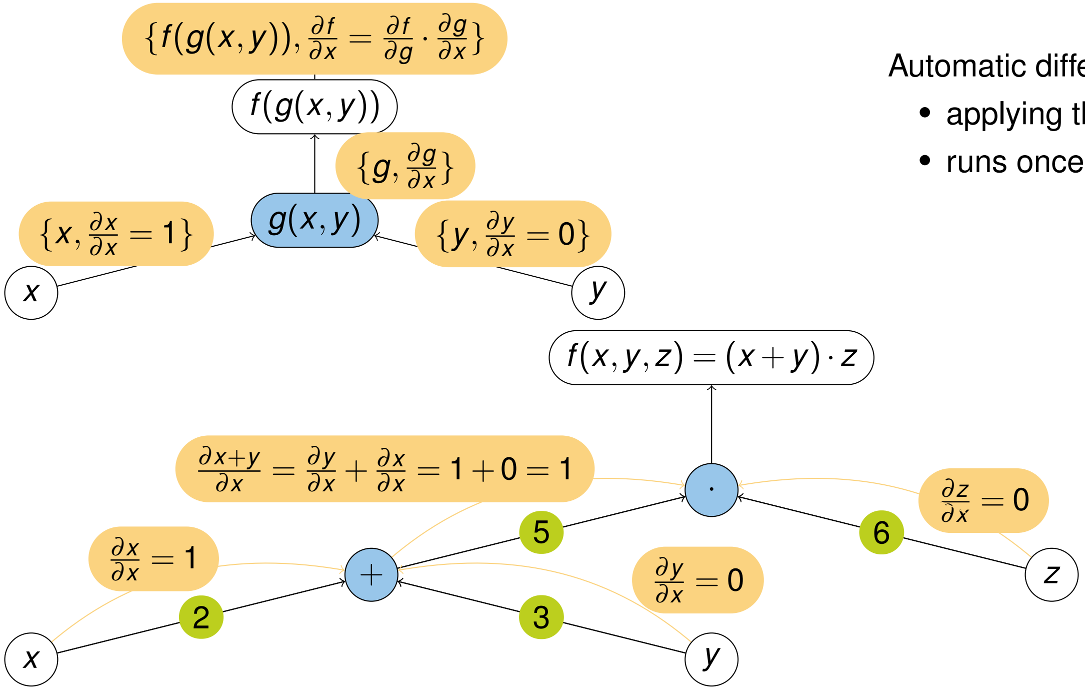

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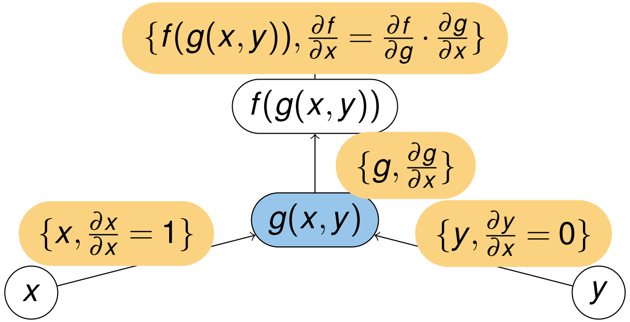

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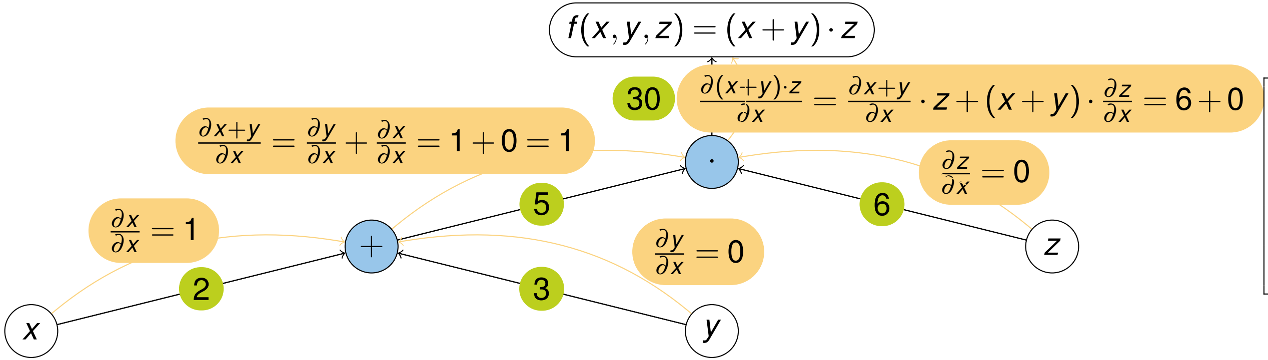


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select * from umbra.derivation2(
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  lambda(x)((x.x+x.y)*x.z));
-- x y z d_x d_y d_z
-- 2 3 6 6 6 5
```

Forward Mode Automatic Differentiation

function EVAL(Z, V)
  if *isVariable*(Z) **then**
    if Z matches V **then return** $\{z, 1\}$
    elsereturn $\{z, 0\}$
  else$\{x, x'\} \leftarrow$ EVAL(X, V); $\{y, y'\} \leftarrow$ EVAL(Y, V)
    if Z matches $X + Y$ **then return** $\{x + y, x' + y'\}$
    else if Z matches $X - Y$ **then return** $\{x - y, x' - y'\}$
    else if Z matches $X \cdot Y$ **then return** $\{x \cdot y, x' \cdot y + x \cdot y'\}$

Input: $(x + y) \cdot z$, $x := 2$, $y := 3$, $z := 6$
eval($(x + y) \cdot z$,x):

```
shared_ptr<dual> forwardDeriveBinaryExpression(BinaryExpression* z, IU* v) {
  switch (z->getFunction()) {
    case KnownFunction::Add: {
      auto left = forwardDeriveExpression(z->getInput(0), v);
      auto right = forwardDeriveExpression(z->getInput(1), v);
      return make_shared<Autodiff::dual>(/*...*/
```

Forward Mode Automatic Differentiation

function EVAL(Z, V)
  if *isVariable*(Z) **then**
    if Z matches V **then return** $\{z, 1\}$
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Input: $(x + y) \cdot z$, $x := 2$, $y := 3$, $z := 6$
eval($(x + y) \cdot z$,x):

- eval($(x + y)$,x):
- eval(z,x): returns {6,0}

```
shared_ptr<dual> forwardDeriveBinaryExpression(BinaryExpression* z, IU* v) {
   switch (z->getFunction()) {
      case KnownFunction::Add: {
         auto left = forwardDeriveExpression(z->getInput(0), v);
         auto right = forwardDeriveExpression(z->getInput(1), v);
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```

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function EVAL(Z, V)
    if *isVariable*(Z) **then**
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eval($(x + y) \cdot z$,x):

- eval($(x + y)$,x):
  - eval(x,x): returns {2,1}
  - eval(y,x): returns {3,0}
- eval(z,x): returns {6,0}

```
shared_ptr<dual> forwardDeriveBinaryExpression(BinaryExpression* z, IU* v) {
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Input: $(x + y) \cdot z$, $x := 2$, $y := 3$, $z := 6$
eval($(x + y) \cdot z$,x):

- eval($(x + y)$,x): returns $\{5, 1 + 0 = 1\}$
  - eval(x,x): returns {2,1}
  - eval(y,x): returns {3,0}
- eval(z,x): returns {6,0}

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```

Forward Mode Automatic Differentiation

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Input: $(x+y) \cdot z$, $x := 2$, $y := 3$, $z := 6$
eval($(x+y) \cdot z$,x): returns {5,$1 \cdot 6 + 5 \cdot 0 = 6$}

- eval($(x+y)$,x): returns {5,$1 + 0 = 1$}
  - eval(x,x): returns {2,1}
  - eval(y,x): returns {3,0}
- eval(z,x): returns {6,0}

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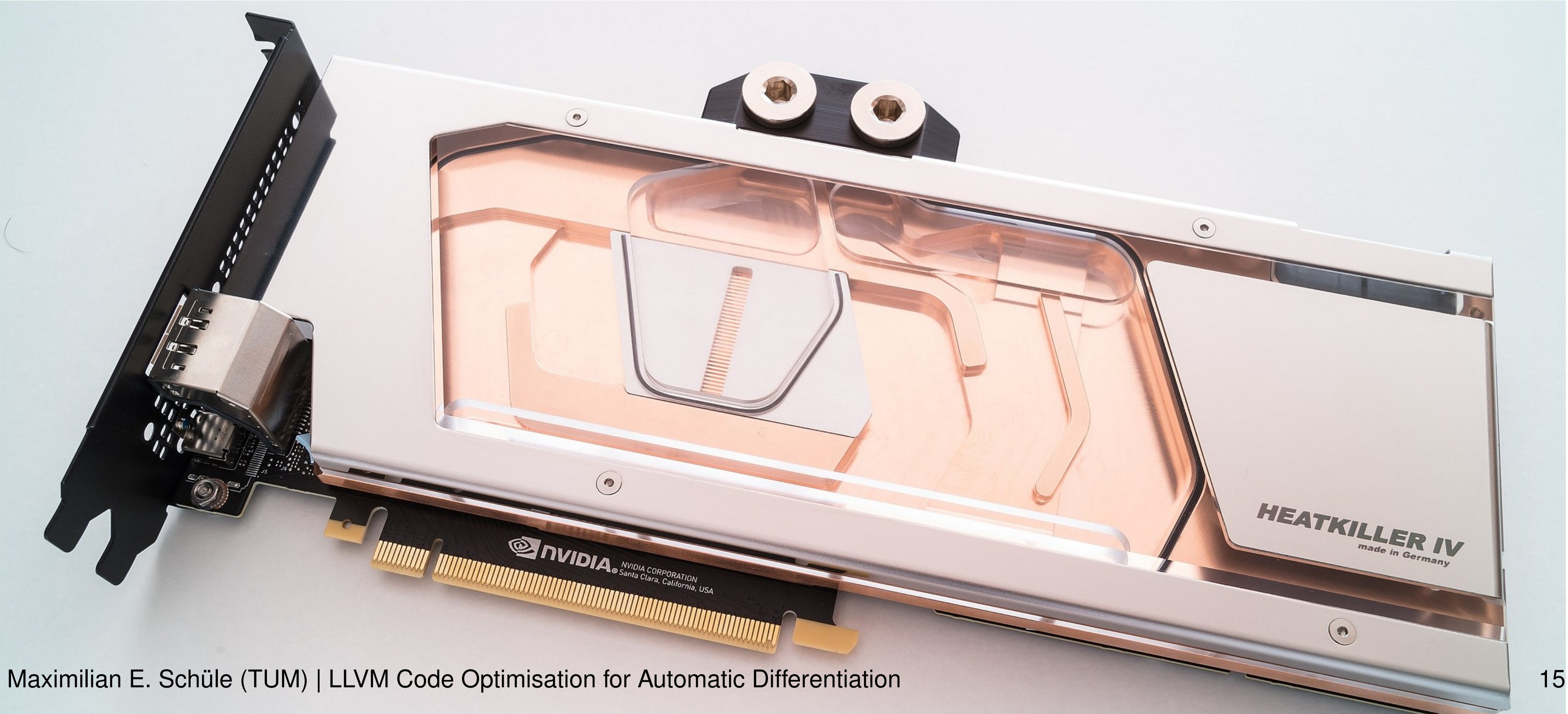
Code-Generation for GPU



Code-Generation for GPU

LLVM IR: forward mode (before compiler optimisation)

- Example: linear regression, mean squared error:

$$L = (w_0 \cdot x_0 + x_1 \cdot w_1 + b - y)^2,$$
$$\frac{\partial L}{\partial w_0} = 2 \cdot (w_0 \cdot x_0 + x_1 \cdot w_1 + b - y) \cdot x_0,$$
$$\frac{\partial L}{\partial w_1} = 2 \cdot (w_0 \cdot x_0 + x_1 \cdot w_1 + b - y) \cdot x_1.$$

- $(w_0 \cdot x_0 + x_1 \cdot w_1 + b - y)$ computed twice
- red: GPU-specific (determine memory positions)

```
%4 = tail call i32 @llvm.nvvm.read.ptx.sreg.tid.x(), !range !8
%5 = tail call i32 @llvm.nvvm.read.ptx.sreg.ntid.x(), !range !9
%6 = tail call i32 @llvm.nvvm.read.ptx.sreg.ctaid.x(), !range !10
%CUDABuiltin_cpp_97_ = mul i32 %6, %5
%CUDABuiltin_cpp_97_0 = add i32 %CUDABuiltin_cpp_97_, %4
%CodeGen_cpp_1539_ = zext i32 %CUDABuiltin_cpp_97_0 to i64
%Autodiff_cpp_700_ = getelementptr double, double* %arg4, i64 %CodeGen_cpp_1539_
%Autodiff_cpp_700_2 = getelementptr double, double* %arg5, i64 %CodeGen_cpp_1539_
%Autodiff_cpp_700_4 = getelementptr double, double* %3, i64 %CodeGen_cpp_1539_
%Autodiff_cpp_700_6 = getelementptr double, double* %2, i64 %CodeGen_cpp_1539_
%Autodiff_cpp_700_8 = getelementptr double, double* %0, i64 %CodeGen_cpp_1539_
%Autodiff_cpp_700_10 = getelementptr double, double* %1, i64 %CodeGen_cpp_1539_
%Autodiff_cpp_519_ = load double, double* %Autodiff_cpp_700_, align 8
%Autodiff_cpp_519_11 = load double, double* %Autodiff_cpp_700_4, align 8
%Autodiff_cpp_519_12 = load double, double* %Autodiff_cpp_700_6, align 8
%Autodiff_cpp_501_ = fmul double %Autodiff_cpp_519_11, %Autodiff_cpp_519_12
%Autodiff_cpp_519_13 = load double, double* %Autodiff_cpp_700_10, align 8
%Autodiff_cpp_519_14 = load double, double* %Autodiff_cpp_700_8, align 8
%Autodiff_cpp_501_15 = fmul double %Autodiff_cpp_519_13, %Autodiff_cpp_519_14
%Autodiff_cpp_493_ = fadd double %Autodiff_cpp_501_, %Autodiff_cpp_501_15
%Autodiff_cpp_493_16 = fadd double %Autodiff_cpp_519_, %Autodiff_cpp_493_
%Autodiff_cpp_519_17 = load double, double* %Autodiff_cpp_700_2, align 8
%Autodiff_cpp_497_ = fsub double %Autodiff_cpp_493_16, %Autodiff_cpp_519_17
%Autodiff_cpp_501_18 = fmul double %Autodiff_cpp_497_, 2.000000e+00
%Autodiff_cpp_501_20 = fmul double %Autodiff_cpp_519_13, %Autodiff_cpp_501_18
%7 = getelementptr inbounds double, double* %result, i64 %CodeGen_cpp_1539_
store double %Autodiff_cpp_501_20, double* %7, align 8
%Autodiff_cpp_519_21 = load double, double* %Autodiff_cpp_700_, align 8
%Autodiff_cpp_519_22 = load double, double* %Autodiff_cpp_700_4, align 8
%Autodiff_cpp_519_23 = load double, double* %Autodiff_cpp_700_6, align 8
%Autodiff_cpp_501_24 = fmul double %Autodiff_cpp_519_22, %Autodiff_cpp_519_23
%Autodiff_cpp_519_25 = load double, double* %Autodiff_cpp_700_10, align 8
%Autodiff_cpp_519_26 = load double, double* %Autodiff_cpp_700_8, align 8
%Autodiff_cpp_501_27 = fmul double %Autodiff_cpp_519_25, %Autodiff_cpp_519_26
%Autodiff_cpp_493_28 = fadd double %Autodiff_cpp_501_24, %Autodiff_cpp_501_27
%Autodiff_cpp_493_29 = fadd double %Autodiff_cpp_519_21, %Autodiff_cpp_493_28
%Autodiff_cpp_519_30 = load double, double* %Autodiff_cpp_700_2, align 8
%Autodiff_cpp_497_31 = fsub double %Autodiff_cpp_493_29, %Autodiff_cpp_519_30
%Autodiff_cpp_501_32 = fmul double %Autodiff_cpp_497_31, 2.000000e+00
%Autodiff_cpp_501_34 = fmul double %Autodiff_cpp_519_22, %Autodiff_cpp_501_32
%Autodiff_cpp_704_ = add i32 %CUDABuiltin_cpp_97_0, 32768
%CodeGen_cpp_1599_35 = zext i32 %Autodiff_cpp_704_ to i64
%8 = getelementptr inbounds double, double* %result, i64 %CodeGen_cpp_1599_35
store double %Autodiff_cpp_501_34, double* %8, align 8
ret void
```

Code-Generation for GPU

LLVM IR: forward mode (after compiler optimisation)

- Example: linear regression, mean squared error:

$$L = (w_0 \cdot x_0 + x_1 \cdot w_1 + b - y)^2,$$
$$\frac{\partial L}{\partial w_0} = 2 \cdot (w_0 \cdot x_0 + x_1 \cdot w_1 + b - y) \cdot x_0,$$
$$\frac{\partial L}{\partial w_1} = 2 \cdot (w_0 \cdot x_0 + x_1 \cdot w_1 + b - y) \cdot x_1.$$

- $(w_0 \cdot x_0 + x_1 \cdot w_1 + b - y)$ computed **once**
- red: GPU-specific (determine memory positions)

```
%3 = tail call i32 @llvm.nvvm.read.ptx.sreg.tid.x(), !range !8
%4 = tail call i32 @llvm.nvvm.read.ptx.sreg.ntid.x(), !range !9
%5 = tail call i32 @llvm.nvvm.read.ptx.sreg.ctaid.x(), !range !10
%CUDABuiltin_cpp_97_ = mul i32 %5, %4
%CUDABuiltin_cpp_97_0 = add i32 %CUDABuiltin_cpp_97_, %3
%CodeGen_cpp_1539_ = zext i32 %CUDABuiltin_cpp_97_0 to i64
%Autodiff_cpp_700_ = getelementptr double, double* %arg4, i64 %CodeGen_cpp_1539_
%Autodiff_cpp_700_2 = getelementptr double, double* %arg5, i64 %CodeGen_cpp_1539_
%Autodiff_cpp_700_4 = getelementptr double, double* %2, i64 %CodeGen_cpp_1539_
%Autodiff_cpp_700_6 = getelementptr double, double* %1, i64 %CodeGen_cpp_1539_
%Autodiff_cpp_700_8 = getelementptr double, double* %0, i64 %CodeGen_cpp_1539_
%Autodiff_cpp_700_10 = getelementptr double, double* %"01", i64 %CodeGen_cpp_1539_
%Autodiff_cpp_519_ = load double, double* %Autodiff_cpp_700_, align 8
%Autodiff_cpp_519_11 = load double, double* %Autodiff_cpp_700_4, align 8
%Autodiff_cpp_519_12 = load double, double* %Autodiff_cpp_700_6, align 8
%Autodiff_cpp_501_ = fmul double %Autodiff_cpp_519_11, %Autodiff_cpp_519_12
%Autodiff_cpp_519_13 = load double, double* %Autodiff_cpp_700_10, align 8
%Autodiff_cpp_519_14 = load double, double* %Autodiff_cpp_700_8, align 8
%Autodiff_cpp_501_15 = fmul double %Autodiff_cpp_519_13, %Autodiff_cpp_519_14
%Autodiff_cpp_493_ = fadd double %Autodiff_cpp_501_, %Autodiff_cpp_501_15
%Autodiff_cpp_493_16 = fadd double %Autodiff_cpp_519_, %Autodiff_cpp_493_
%Autodiff_cpp_519_17 = load double, double* %Autodiff_cpp_700_2, align 8
%Autodiff_cpp_497_ = fsub double %Autodiff_cpp_493_16, %Autodiff_cpp_519_17
%Autodiff_cpp_501_18 = fmul double %Autodiff_cpp_497_, 2.000000e+00
%Autodiff_cpp_501_20 = fmul double %Autodiff_cpp_519_13, %Autodiff_cpp_501_18
%6 = getelementptr inbounds double, double* %result, i64 %CodeGen_cpp_1539_
store double %Autodiff_cpp_501_20, double* %6, align 8
%Autodiff_cpp_501_34 = fmul double %Autodiff_cpp_519_11, %Autodiff_cpp_501_18
%Autodiff_cpp_704_ = add i32 %CUDABuiltin_cpp_97_0, 32768
%CodeGen_cpp_1599_35 = zext i32 %Autodiff_cpp_704_ to i64
%7 = getelementptr inbounds double, double* %result, i64 %CodeGen_cpp_1599_35
store double %Autodiff_cpp_501_34, double* %7, align 8
ret void
```

Code-Generation for GPU

LLVM IR: reverse mode

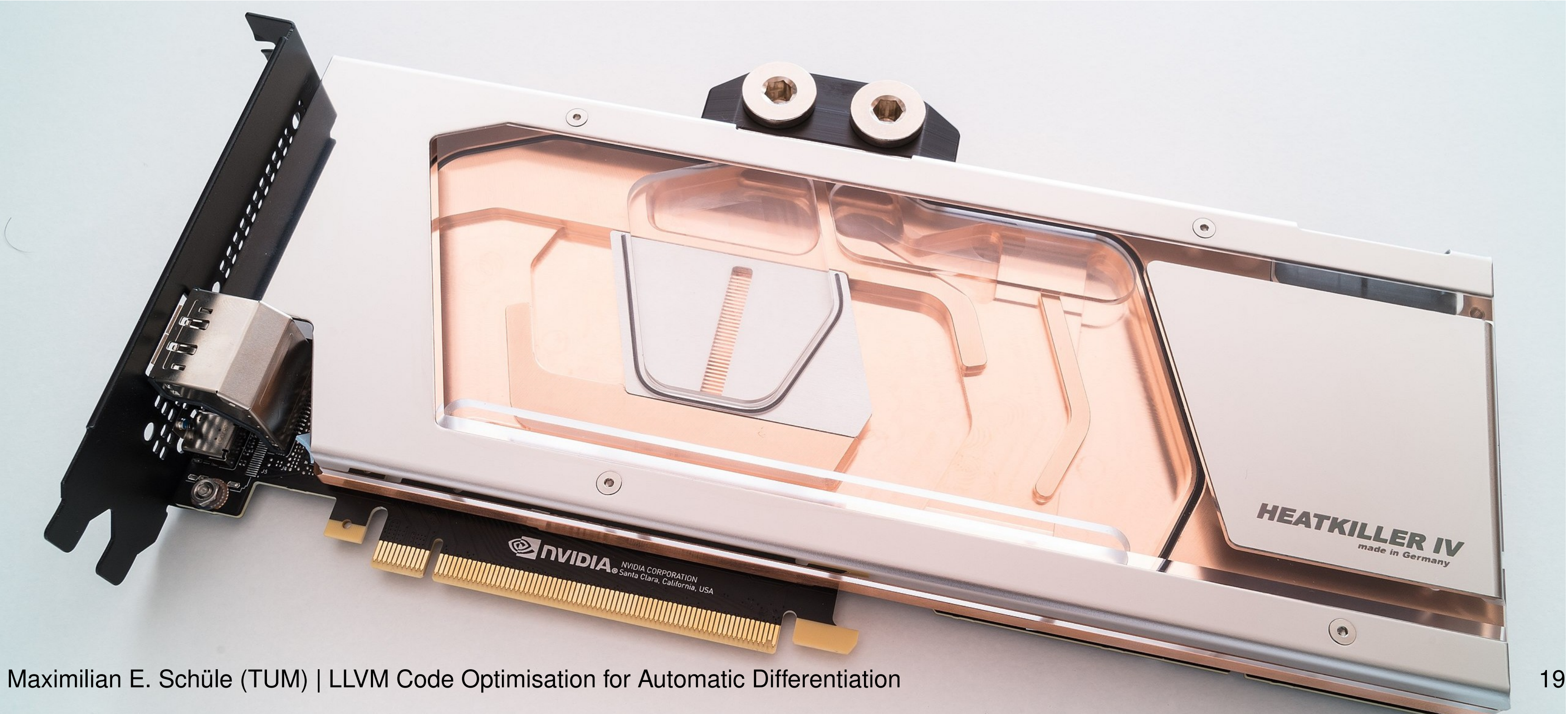
- Example: linear regression, mean squared error:

$$L = (w_0 \cdot x_0 + x_1 \cdot w_1 + b - y)^2,$$
$$\frac{\partial L}{\partial w_0} = 2 \cdot (w_0 \cdot x_0 + x_1 \cdot w_1 + b - y) \cdot x_0,$$
$$\frac{\partial L}{\partial w_1} = 2 \cdot (w_0 \cdot x_0 + x_1 \cdot w_1 + b - y) \cdot x_1.$$

- $(w_0 \cdot x_0 + x_1 \cdot w_1 + b - y)$ computed **once**
- red: GPU-specific (determine memory positions)

```
%4 = tail call i32 @llvm.nvvm.read.ptx.sreg.tid.x(), !range !8
%5 = tail call i32 @llvm.nvvm.read.ptx.sreg.ntid.x(), !range !9
%6 = tail call i32 @llvm.nvvm.read.ptx.sreg.ctaid.x(), !range !10
%CUDABuiltin_cpp_97_ = mul i32 %6, %5
%CUDABuiltin_cpp_97_1 = add i32 %CUDABuiltin_cpp_97_, %4
%CodeGen_cpp_1539_ = zext i32 %CUDABuiltin_cpp_97_1 to i64
%Autodiff_cpp_616_ = getelementptr double, double* %arg0, i64 %CodeGen_cpp_1539_
%Autodiff_cpp_616_3 = getelementptr double, double* %arg00, i64 %CodeGen_cpp_1539_
%Autodiff_cpp_616_5 = getelementptr double, double* %3, i64 %CodeGen_cpp_1539_
%Autodiff_cpp_616_7 = getelementptr double, double* %2, i64 %CodeGen_cpp_1539_
%Autodiff_cpp_616_9 = getelementptr double, double* %0, i64 %CodeGen_cpp_1539_
%Autodiff_cpp_616_11 = getelementptr double, double* %1, i64 %CodeGen_cpp_1539_
%Autodiff_cpp_519_ = load double, double* %Autodiff_cpp_616_, align 8
%Autodiff_cpp_519_12 = load double, double* %Autodiff_cpp_616_5, align 8
%Autodiff_cpp_519_13 = load double, double* %Autodiff_cpp_616_7, align 8
%Autodiff_cpp_501_ = fmul double %Autodiff_cpp_519_12, %Autodiff_cpp_519_13
%Autodiff_cpp_519_14 = load double, double* %Autodiff_cpp_616_11, align 8
%Autodiff_cpp_519_15 = load double, double* %Autodiff_cpp_616_9, align 8
%Autodiff_cpp_501_16 = fmul double %Autodiff_cpp_519_14, %Autodiff_cpp_519_15
%Autodiff_cpp_493_ = fadd double %Autodiff_cpp_501_, %Autodiff_cpp_501_16
%Autodiff_cpp_493_17 = fadd double %Autodiff_cpp_519_, %Autodiff_cpp_493_
%Autodiff_cpp_519_18 = load double, double* %Autodiff_cpp_616_3, align 8
%Autodiff_cpp_497_ = fsub double %Autodiff_cpp_493_17, %Autodiff_cpp_519_18
%Autodiff_cpp_317_19 = fmul double %Autodiff_cpp_497_, 2.000000e+00
%Autodiff_cpp_302_ = fmul double %Autodiff_cpp_519_14, %Autodiff_cpp_317_19
%Autodiff_cpp_302_23 = fmul double %Autodiff_cpp_519_12, %Autodiff_cpp_317_19
%7 = getelementptr inbounds double, double* %result, i64 %CodeGen_cpp_1539_
store double %Autodiff_cpp_302_, double* %7, align 8
%Autodiff_cpp_623_ = add i32 %CUDABuiltin_cpp_97_1, 32768
%CodeGen_cpp_1599_26 = zext i32 %Autodiff_cpp_623_ to i64
%8 = getelementptr inbounds double, double* %result, i64 %CodeGen_cpp_1599_26
store double %Autodiff_cpp_302_23, double* %8, align 8
ret void
```

Evaluation


NVIDIA
NVIDIA CORPORATION
Santa Clara, California, USA
HEATKILLER IV
made in Germany

Evaluation: Comparison Forward/Reverse Mode

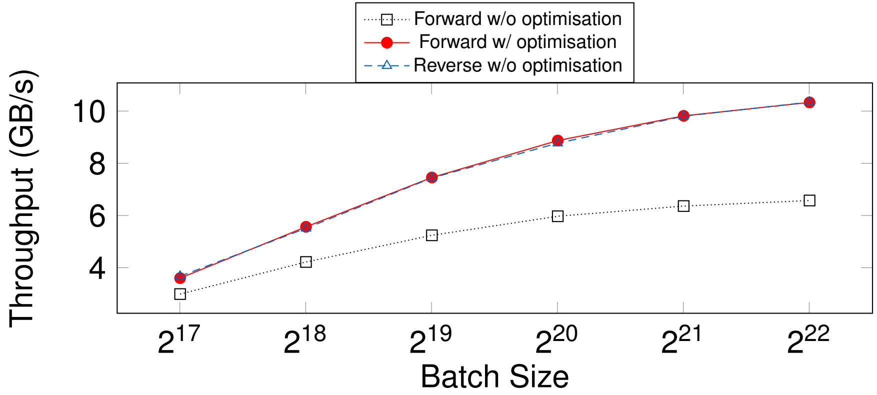


Figure: Execution time.

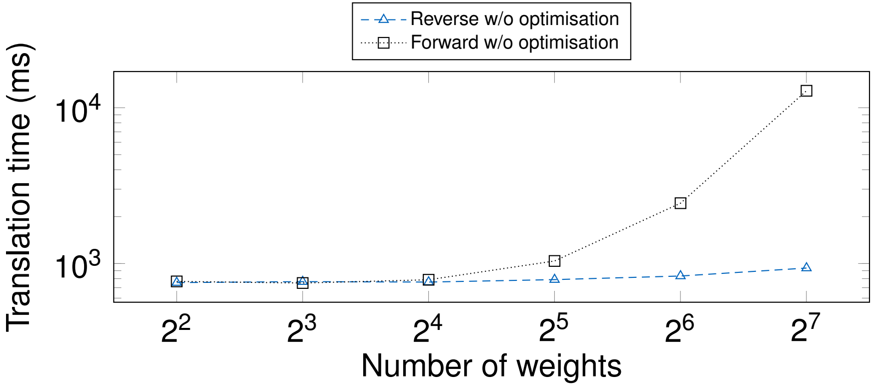
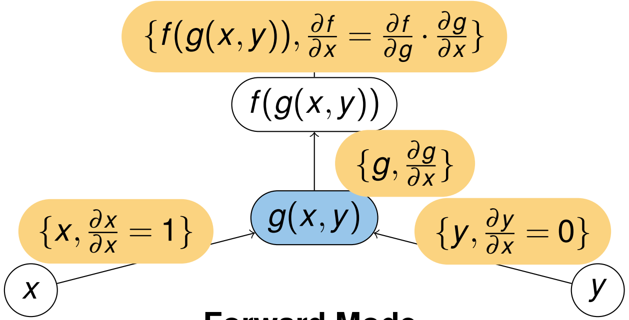


Figure: Compilation time (1 thread).

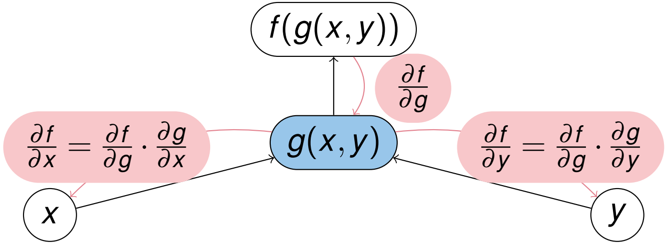
- *System*: NVIDIA GeForce RTX 3050 Ti Laptop, Intel Core i7-11800H.
- *Runtime*: noalias optimises the generated code, forward and reverse mode indeed lead to the same performance
- *Compilation time*: depends on number of variables

Conclusion



Forward Mode

- compilation time dependent on number of derivatives
- optimised execution time similar to reverse mode



Reverse Mode

- runtime and compilation time do not depend on number of derivatives
- well suited without optimisation

Future Work

- Automatic differentiation to SQL: use reverse mode
- Performance comparison also when training neural networks

Thank you for your attention!